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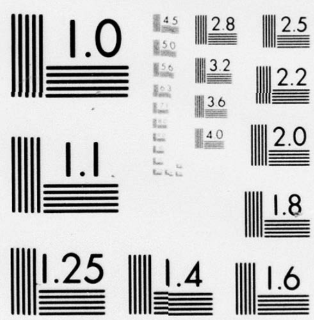
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STABILITY CRITERIA FOR CONTINUOUS-TIME STEMS WITH

COLORED MULTIPLICATIVE NOISE

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ABSTRACT

Uncertain elements must be considered in the mathematical model of many dynamical systems. The theory associated with additive noise models is quite advanced, but many control problems are more realistically modeled as containing uncertain and time varying parameters or gains. For multiplicative gains of the white noise type, necessary and sufficient conditions for the stability of many of these systems have been derived previously. In this article, we develop conditions for the stability of some continuous-time systems containing multiplicative colored noise.

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I. INTRODUCTION:

In this article we analyze some continuous-time systems that contain uncertain parameters. The statistical properties of the uncertain elements are used to determine conditions that will guarantee the stability (in some probabilistic sense) of these systems. By far, the bulk of the research completed on this topic concerns systems in which the noise terms enter additively as control or observation noise. Systems in which the noise enters as a multiplier have been given more attention lately, but the results obtained so far are quite incomplete. In the following sections, we make some contributions to this theory.

For linear systems in which the random process enters additively, the use of a white noise model is well justified, especially if state augmentation is used to generate the colored noise processes required. Even without state augmentation, the white noise model can often be used to represent wideband processes because the extra power in the white noise model will be dissipated anyway. For multiplicative noise, however, the situation is not so simple. Using state augmentation and white noise to generate the required colored noise process leads to a <u>non-linear</u> system model which is, in fact, <u>bilinear</u>. Without state augmentation, the only interpretation of the results is in terms of physical systems involving very wideband noise processes. The main contribution of this research is the establishment of some criteria for the stability of the systems described below that explicitly involve the power and the bandwidth of the colored noise.

In the following sections of this article, an analysis progresses from first order systems to higher order systems. Sufficient and sometimes necessary conditions for the exponential mean square stability of the null solution are derived. In Section IV, the damped harmonic oscillator problem is treated in some detail, and sufficient conditions for the mean square stability of its null solution are given. The damped harmonic oscillator problem was chosen because it has received considerable attention in the literature [1, 2] so results are available for comparison. It is an example of a system that does not evolve on a solvable Lie group, and we are not able to duplicate the elegant results that are often obtainable in those cases.

II. SYSTEM DEFINITION:

We will be studying systems of the form:

$$\frac{dx(t)}{dt} = [A + f(t)B]\, x(t) \tag{1}$$

The initial condition is a random variable, A and B are constant matrices, and $f(\bullet)$ is a real-valued colored noise process. The easiest example to visualize is a linear dynamical system containing a noisy gain in the feedback path, as in Figure (1).

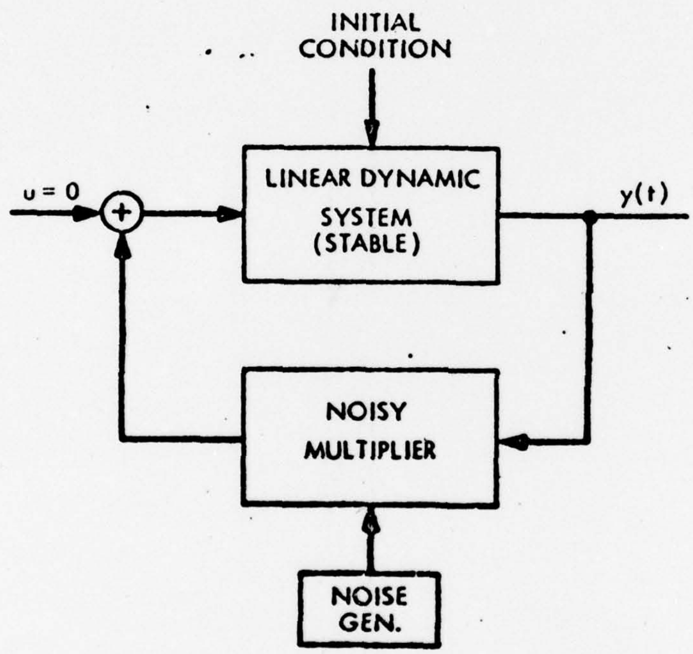

FIGURE 1

MODEL OF LINEAR DYNAMICAL SYSTEM WITH NOISY FEEDBACK GAIN

One major difficulty in studying and comparing the results found in the literature is the abundance of concepts and definitions surrounding the phrase "stochastic stability". To avoid any problems of this kind, only the following types of stability will be considered here:

Definition:

The zero equilibrium solution of system (1) is said to possess exponential stability of the pth mean if there exists positive constants α, β, and δ, such that $|x(o)| < \delta$ implies that for all $t > 0$,

$$E\left\{\left|x(t)\right|_p^p\right\} \leq \beta\left|x(0)\right|_p^p \exp(-\alpha t) \qquad (2)$$

where

$$\left|x(t)\right|_p^p = \sum_{i=1}^{n}\left|x_i(t)\right|^p$$

In this article, the case $p = 2$ will receive the most consideration and the phrase "exponentially stable in the mean square" will be used.

III. SCALAR, FIRST ORDER SYSTEMS:

Although first order systems represent only a small fraction of all interesting dynamical systems, they play an important role in the research described below for two reasons. The formulae can be evaluated easily and provide an example to follow in the higher dimensional cases, and the formulae found for the first order case lead to a bound for the mean square stability of higher order systems. The detailed consideration of this case seems warranted.

Colored Noise Results:

It turns out that one can solve first order linear stochastic differential equations because the solution can be expressed in terms of a functional of the random processes involved. [3] That is, suppose f(t) is a scalar function of time. Then the equation

$$\dot{x} = (a + f(t))x \qquad x(0) = x_0 \tag{3}$$

has for its solution

$$x(t) = x_0 \exp \left(\int_0^t (a + f(\sigma))d\sigma \right) \tag{4}$$

Moreover, the mean square of x(t) is given by:

$$E\{x(t)x^*(t)\} = E[\exp(2at + 2\int_0^t f(\sigma)d\sigma)]x_o^2$$

$$= e^{2at}\, E[\exp(2\int_0^t f(\sigma)d\sigma)]x_o^2 \tag{5}$$

Furthermore, if f(t) is a Gaussian colored noise process, one can calculate all of the statistical properties of the exponential function in (4).

For the purpose of making comparisons with later results, conditions that guarantee exponential mean square stability are derived next. Assume now that f(t) is a real-valued zero mean, stationary Gaussian random process with autocorrelation function $R_f(t_1, t_2) = \sigma_f^2\, e^{-\alpha|t_1-t_2|}$, and define the process

$$\eta(t) = \int_0^t f(\sigma)d\sigma$$

Then $\eta(t)$ is a Gaussian random process with zero mean and autocorrelation function given by:

$$R_\eta(t_1, t_2) = R_f(t_1, t_2) * h(t_1) * h(t_2) \tag{6}$$

where h(t) is the impulse response of an integrator. $\eta(t)$ is not stationary, for at the very least there are initial transients in the moments of the process.

Performing the indicated convolutions in Equation (6), and setting $t_1 = t_2 = t$, one obtains:

$$R_\eta(t, t) = \frac{2}{\alpha} \sigma_f^2 t + \frac{\sigma_f^2}{\alpha^2} [2e^{-\alpha t} - 2], \quad t \geq 0$$

Knowing Equation (4), it is straightforward to calculate criteria for the exponential stability of the pth mean of the null solution of System (3). A special case ($p = 1$) is treated by Brockett [4, page 58].

Clearly,

$$x^p(t) = x^p(0) \exp [p \int_0^t (a + f(\sigma))d\sigma] \tag{7}$$

so

$$E\{|x(t)|^p\} = e^{pat} E\{e^{p\eta(t)}\}|x_o|^p \tag{8}$$

But, $\eta(t)$ is Gaussian, so

$$E\{e^{p\eta(t)}\} = e^{(p^2/2)R_\eta(t,t)}$$

and the null solution of System (3) is exponentially stable in the pth mean if and only if

$$a + p \frac{\sigma_f^2}{\alpha} < 0 \tag{9}$$

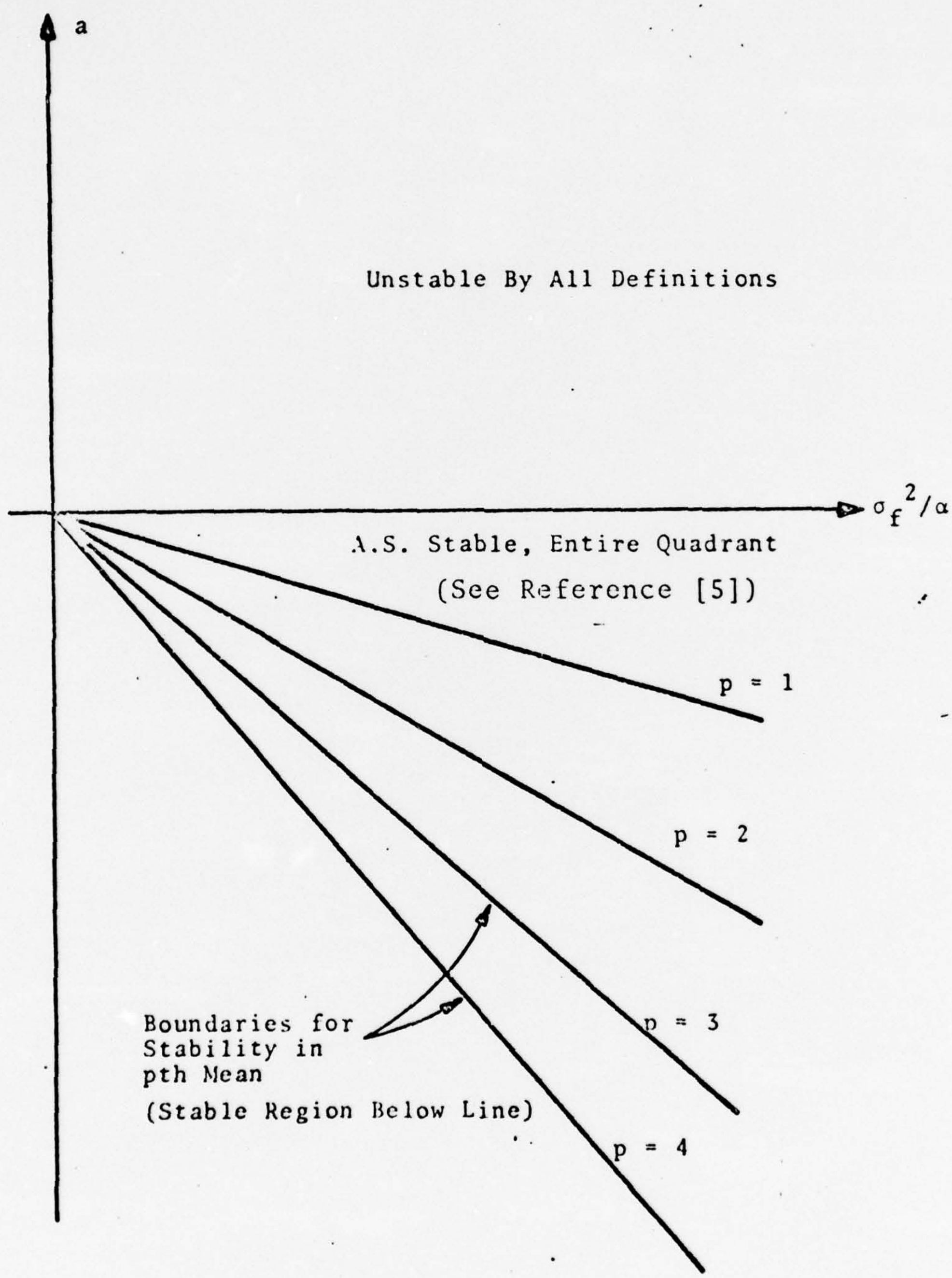

FIGURE 2

STABILITY REGIONS FOR SYSTEM (3)

Referring to Figure (2), it is easy to identify systems that have rather peculiar properties. That is, in the region $0 > a > -\sigma_f^2/\alpha$, System (3) is unstable in the pth mean for $p \geq 1$, yet almost every sample path has been shown to approach zero after large time intervals [5]. Moreover, for any combination of a, σ_f^2 and $\alpha > 0$, System (3) will be unstable in the pth mean if p is chosen large enough. Similarly, for any given p, σ_f^2, and $\alpha > 0$, System (3) will be stable in the pth mean if α is chosen large enough. This last observation has an obvious interpretation; for a given feedback noise power, concentrating that power near zero frequency provides the most destablizing (in pth mean) influence. For further comments on the tradeoffs of studying the behavior of sample paths (almost sure stability criteria) versus pth mean stability, see Kozin [6, 7].

These results are easily generalized to the case in which f(t) is a band-pass process, whose autocorrelation function is given by:

$$R_f(t_1, t_2) = \sigma_f^2 \exp(-\alpha|t_1-t_2|) \cos \omega_o(t_1 - t_2)$$

Once again, the autocorrelation function of $\eta(t)$ is given by:

$$R_\eta(t_1,t_2) = \int_0^{t_1} \int_0^{t_2} R_f(\sigma_1,\sigma_2)d\sigma_1,d\sigma_2$$

$$= \frac{2\sigma_f^2}{\alpha^2 + \omega_o^2} \min(t_1,t_2) + \begin{pmatrix}\text{terms that grow} \\ \text{less than linearly} \\ \text{in } t_1 \text{ or } t_2\end{pmatrix}$$

Then, as in Equation (8),

$$E[x^p(t)] = e^{pat}\, E\{e^{p\eta(t)}\}\, x_o^{\,p}$$
$$= e^{pat}\, e^{(p^2/2)R_\eta(t,t)}\, x_o^{\,p}$$

so

$$\lim_{t\to\infty} E[x^p(t)]=0 \Longleftrightarrow pa + (p^2/2)\left(\frac{2}{\alpha}\,\sigma_f^{\,2}\,\frac{1}{1 + \omega_o^{\,2}/\alpha^2}\right) < 0 \qquad (10)$$

in which case the convergence will be at an exponential rate.

IV. HIGHER ORDER SYSTEMS:

In Reference [8], Rabotnikov uses Picard expansions to derive a criterion for the mean square stability of a single-input single-output linear dynamical equation containing a white noise parameter. The strength of his method lies in the fact that the white noise hypothesis is not used until the end of his proof, and then only to solve some integrals involving the noise autocorrelation function. Without the white noise assumption, the series of integrals obtained is too hard to evaluate. However, the example studied in Section III provides a useful bound for this series of integrals and leads to a simple stability criterion for the example treated in this section.

Above, we derived a criterion for the exponential mean square stability of System (3). Suppose now that we try to derive that same criterion using the Picard expansion of the solution.

Let Z(t) be the solution of the <u>deterministic</u> system

$$Z(t) = - a\, Z(t) \qquad (11)$$

$$Z(0) = x_o$$

Equation (3) can be rewritten as

$$x(t) = Z(t) + \int_0^t w(t - \sigma) f(\sigma) x(\sigma) d\sigma \qquad (12)$$

where $w(t) = e^{-at}$, the impulse response of System (11).

Repeatedly substituting Equation (12) back into itself yields:

$$\begin{aligned} x(t) &= Z(t) + \int_0^t e^{-a(t-\sigma)} f(\sigma) Z(\sigma) d\sigma + \int_0^t e^{-a(t-\sigma)} f(\sigma) \\ &\quad \int_o^\sigma e^{-a(\sigma-q)} f(q) x(q) dq d\sigma \\ &= Z(t) + \int_o^t e^{-a(t-q)} f(q) Z(q) dq + \int_o^t \int_o^{q_1} e^{-a(t-q_1)} \\ &\quad e^{-a(q_1-q_2)} f(q_1) f(q_2) Z(q_2) dq_2 dq_1 \\ &\quad + \int_o^t \int_o^{q_1} \int_o^{q_2} e^{-a(t-q_1)} e^{-a(q_1-q_2)} e^{-a(q_2-q_3)} f(q_1) f(q_2) \\ &\quad f(q_3) Z(q_3) dq_3 dq_2 dq_1 + \dots \end{aligned} \qquad (13)$$

(The convergence of this series is implied by the analysis of Reference [5], Section 2.1.)

Next, by multiplying the series in Equation (13) by itself, $E\{x^2(t)\}$ can be expressed as:

$$
\begin{aligned}
E\{x^2(t)\} &= Z^2(t) + \text{terms involving } E\{f(t)\} \\
&+ E\int_0^t\int_0^t e^{-a(t-q)}e^{-a(t-p)}f(q)f(p)Z(q)Z(p)\,dq\,dp \\
&+ 2Z(t)\,E\int_0^t\int_0^{q_1} a^{-a(t-q_1)}e^{-a(q_1-q_2)}f(q_1)f(q_2)Z(q_2)\,dq_2\,dq_1 \\
&+ \text{terms involving third moments of } f(t) \\
&+ 2Z(t)\,E\int_0^t dq_1\int_0^{q_1} dq_2\int_0^{q_2} dq_3\int_0^{q_3} dq_4\; e^{-a(t-q_1)}e^{-(q_1-q_2)} \\
&\quad e^{-a(q_2-q_3)}e^{-a(q_3-q_4)}f(q_1)f(q_2)f(q_3)f(q_4)Z(q_4) \\
&+ 2E\int_0^t dq_1\int_0^t dp_1\int_0^{q_1} dq_2\int_0^{q_2} dq_3\; e^{-a(t-q_1)}e^{-a(t-p_1)} \\
&\quad e^{-a(q_1-q_2)}e^{-a(q_2-q_3)}f(p_1)f(q_1)f(q_2)f(q_3)Z(q_3)Z(p_1) \\
&+ E\int_0^t dq_1\int_0^{q_1} dq_2\int_0^t dp_1\int_0^{p_1} dp_2\; e^{-a(t-p_1)}e^{-a(t-q_1)} \\
&\quad e^{-a(q_1-q_2)}e^{-a(p_1-p_2)}f(p_1)f(p_2)f(q_1)f(q_2)Z(q_2)Z(p_2) \\
&+ \cdots \qquad\qquad (14)
\end{aligned}
$$

Equations (14) and (8) both give expressions for $E\{x^2(t)\}$, so the right hand side of Equation (8) is an evaluation of the series given in (14). This is important because the series expansion (similar to Equation (14)) for the second order system considered next can be bounded termwise by the right hand side of Equation (14). Therefore, Equation (9) can be employed as a stability criterion for the second order system, but now only as a sufficient condition.

For the remainder of this section, we will be considering the system

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -2\zeta \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} g(t)x_1(t) \qquad (15)$$

$$x(0) = x_o$$

$$y(t) = [0,\ 1]\ x(t) \qquad (16)$$

Also we will assume that we are interested only in the stability of $x_1(t)$. From System (15) we will define the matrices

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2\zeta \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad c = [1,\ 0] \qquad (17)$$

It is also convenient to define

$$r(t) = e^{At}\ x_o \qquad (18)$$

and let g(t) be a stationary colored noise process, Gaussian with zero mean, variance σ_g^2 and autocorrelation function

$$R_g(\tau) = \sigma_g^2 e^{-\alpha|\tau|} \tag{19}$$

Combining Equations (15) through (18), $x_1(t)$ is given by the integral equation

$$x_1(t) = r_1(t) + \int_0^{\tau} w_\zeta(t-\sigma)g(\sigma)x_1(\sigma)d\sigma \tag{20}$$

where

$$w_\zeta(t) = ce^{At}b = \frac{e^{-\zeta t}}{2\sqrt{\zeta^2-1}}\left(e^{\sqrt{\zeta^2-1}\,t} - e^{-\sqrt{\zeta^2-1}\,t}\right) \tag{21}$$

Substituting Equation (20) into itself and squaring yields

$$E\{x_1(t)\} = r_1^2(t) + \text{terms involving first moments of g}$$

$$+ E\int_0^t\int_0^t w_\zeta(t-q)w_\zeta(t-p)g(q)g(p)r_1(q)r_1(p)dqdp$$

$$+ 2r_1(t)\, E\int_0^t\int_0^{q_1} w_\zeta(t-q_1)w_\zeta(q_1-q_2)g(q_1)g(q_2)r_1(q_2)dq_2dq_1$$

$$+ \text{Higher order terms as in Equation (14)} \tag{22}$$

Turning our attention back to Equation (14), and bringing the expected value operator inside the integrals in that equation, we see that every factor of every term is positive. This allows us to prove the following theorem.

Theorem 1:

Consider System (15). Suppose there exist two functions of ζ, say $\beta(\zeta)$ and $\gamma(\zeta)$ and a constant $\delta > 0$, such that

$$r_1(t) \leq \delta\, e^{-\beta(\zeta)t} \qquad t \in [0, \infty] \tag{23}$$

and

$$w_\zeta(t) \leq \gamma(\zeta)\exp(-\beta(\zeta)t) \qquad t \in [0, \infty] \tag{24}$$

Then,

$$\lim_{t\to\infty} E\{x_1^{\,2}(t)\} \to 0 \quad \text{if} \quad \frac{2\sigma_g^{\,2}}{\alpha} < \frac{+\beta(\zeta)}{\gamma^2(\zeta)} \tag{25}$$

and the convergence will be at an exponential rate, with time constant $-\beta(\zeta) + (2/\alpha)\sigma_g^{\,2}\,\gamma^2(\zeta)$.

Proof:

Using Equations (23) and (24) in (22) yields

$$E\{x_1^{\,2}(t)\} \leq \delta^2 Z^2(t) + \text{terms involving first moments of } g$$

$$+ \delta^2 E\int_0^t\int_0^t \gamma^2(\zeta)e^{-\beta(\zeta)(t-q)}e^{-\beta(\zeta)(t-p)}g(q)g(p)Z(q)Z(p)\,dq\,dp$$

$$+ \delta^2\, 2Z(t)E\int_0^t\int_0^{q_1}\gamma^2(\zeta)e^{-\beta(\zeta)(t-q_1)}e^{-\beta(\zeta)(q_1-q_2)}g(q_1)$$

$$g(q_1)g(q_2)Z(q_2)\,dq_2\,dq_1 + \left(\begin{matrix}\text{Higher order terms as}\\ \text{in Equation (14)}\end{matrix}\right) \tag{26}$$

The factor δ^2 does not influence the result, because the limit of the right hand side will be shown to be equal to zero.

Let us now assume the following relations between the parameters in Equation (14) and those in Equation (26). Let

$$f(t) = \gamma(\zeta)g(t) \qquad \sigma_f^{\,2} = \gamma^2(\zeta)\sigma_g^{\,2} \tag{27}$$

$$\beta(\zeta) = + a$$

so that the right hand sides of Equations (26) and (14) are the same except for the factor δ^2. Therefore, Equation (9), which says

$$\lim_{t\to\infty} E\{x^2(t)\} = 0 \quad \text{if} \; - a + \frac{2\sigma_f^{\,2}}{\alpha} < 0,$$

must imply that

$$\lim_{t\to\infty} E\{x_1^{\,2}(t)\} = 0 \text{ if } -\beta(\zeta) + \frac{2}{\alpha}\,\sigma_g^{\,2}\,\gamma^2(\zeta) < 0 \tag{28}$$

This completes the proof, and we now direct our attention to making judicious choices for $\beta(\zeta)$ and $\gamma(\zeta)$.

Optimizing the Choice of β and γ:

There are conditions that $\beta(\zeta)$ and $\gamma(\zeta)$ must satisfy as functions of ζ. Therefore, we are free to choose $\beta(\zeta)$ and $\gamma(\zeta)$ independently for each value of ζ. The following constrained optimization problem can, therefore, be solved in order to optimize the strength of Theorem 1.

$$\underset{\beta,\ \gamma}{\text{Maximize}} \quad \frac{\beta}{\gamma^2} \quad \text{subject to } w_\zeta(t) \leq \gamma \exp(-\beta t) \text{ for all } t \in [0, \infty]. \tag{29}$$

The values of β and γ at which the maximum is achieved for each value of ζ form the functions $\beta(\zeta)$ and $\delta(\zeta)$. Using these values of $\beta(\zeta)$ and $\delta(\zeta)$ leads to the strongest version of Theorem 1. The maximization procedure has been accomplished with a small computer program which produced the graphs given in Figure 3.

In the literature [9, 10, 6] there have been a series of stability boundaries derived for System (15) that apply to the colored noise case, but none of which depend upon the bandwidth of the noise. The boundary derived by Infante [1] is the best of those reported, and is superimposed onto Figure 3. For very low ζ, the curves given by Equation (28) are parabolic, and, hence, greater than the Infante curve for any value of α. However, the Infante boundary is comparable or better than Theorem 1 for values of ζ up to about 8. For wider bandwidths, the new boundary is superior, except for ζ corresponding to greatly overdamped systems. Also, our bound considers only $E\{x_1^2(t)\}$, whereas Infante has used $E\{x_1^2(t) + x_2^2(t)\}$.

In the literature [7], there have appeared plots of the mean square stability criteria for System (15) superimposed on the criteria for the corresponding white noise parameter system (Itô sense). This practice is misleading because, in the

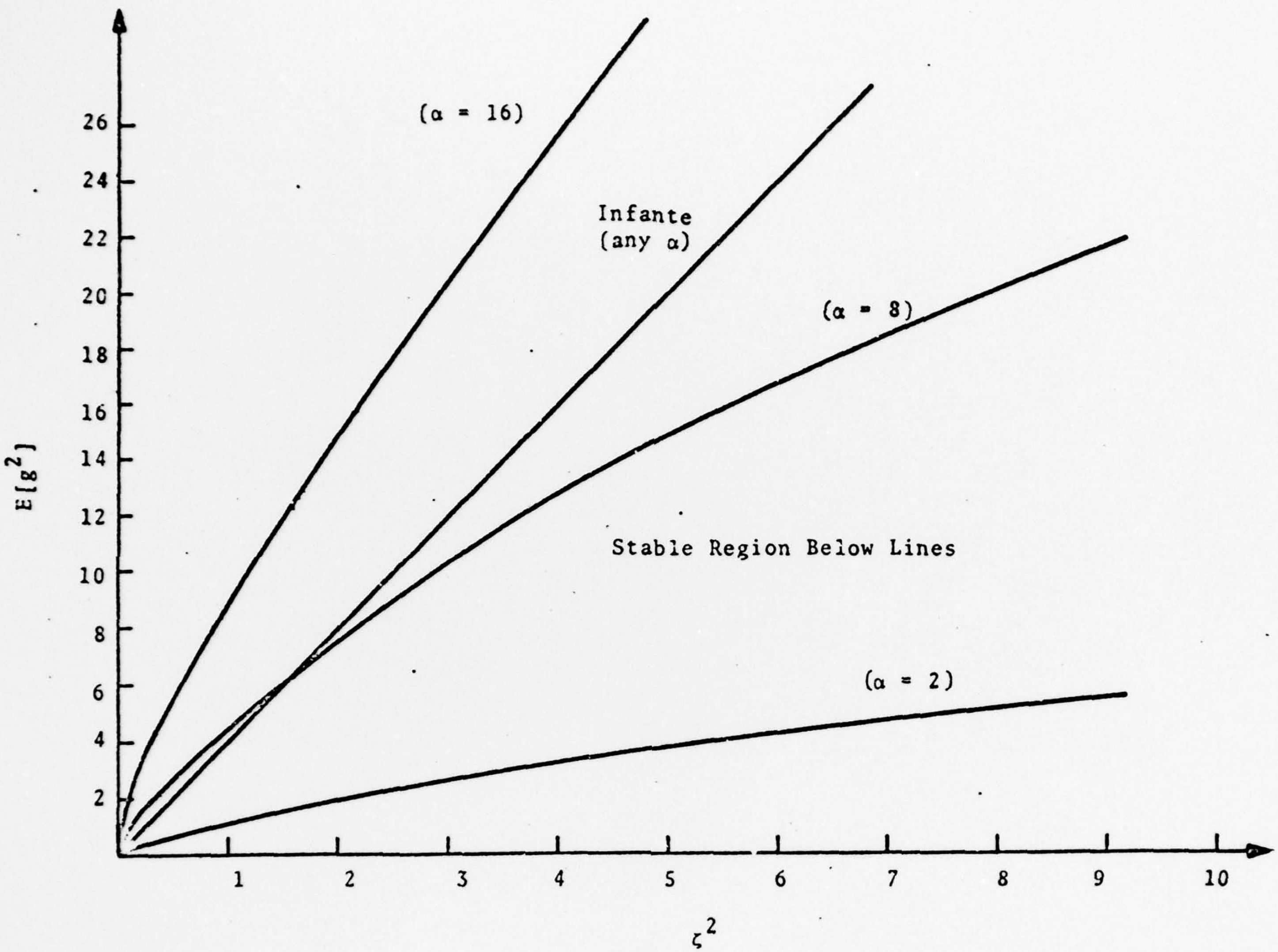

FIGURE 3

STABILITY REGIONS FOR SYSTEM (15) OPTIMIZED BY EQUATION (29)

Infante case, the boundary is given for $E\{g^2(t)\}$. In the Itô case, the bound is given in terms of σ_g^2, where g(t) is white noise with autocorrelation function

$$R_g(\tau) = \sigma_g^2 \, \delta(\tau)$$

In the following paragraphs the result of Theorem 1 is recalculated in such a way that the comparison to the Itô result is justified.

The power spectral density of g(t) corresponding to Equation (19) is given by:

$$S_g(\omega) = \frac{(2/\alpha)\,\sigma_g^2}{[1 + (\omega^2/\alpha^2)]}$$

Suppose now that we change the vertical axis on Figure (3), so that the stability regions are plotted in terms of $S_g(0)$. If we do this, all the graphs scale into the same graph, which is independent of α. This procedure leads to Figure (4). The data presented in this way indicates how the region of stability changes as $\alpha \to \infty$. It does not change! It is known that there are no Itô correction terms required for System (15), so the boundary in Figure 4 is also a boundary in the limit as $\alpha \to \infty$, i.e., the white noise case. Superimposed on Figure 4 is the necessary and sufficient condition for the mean square stability of System (15) for the white noise case [2]. We see that as ζ^2 becomes large, the boundary of Equation (28) differs from the Itô result by just a factor of 2. It is reasonable to expect the two answers to differ by such an amount because the inequality (24) is really

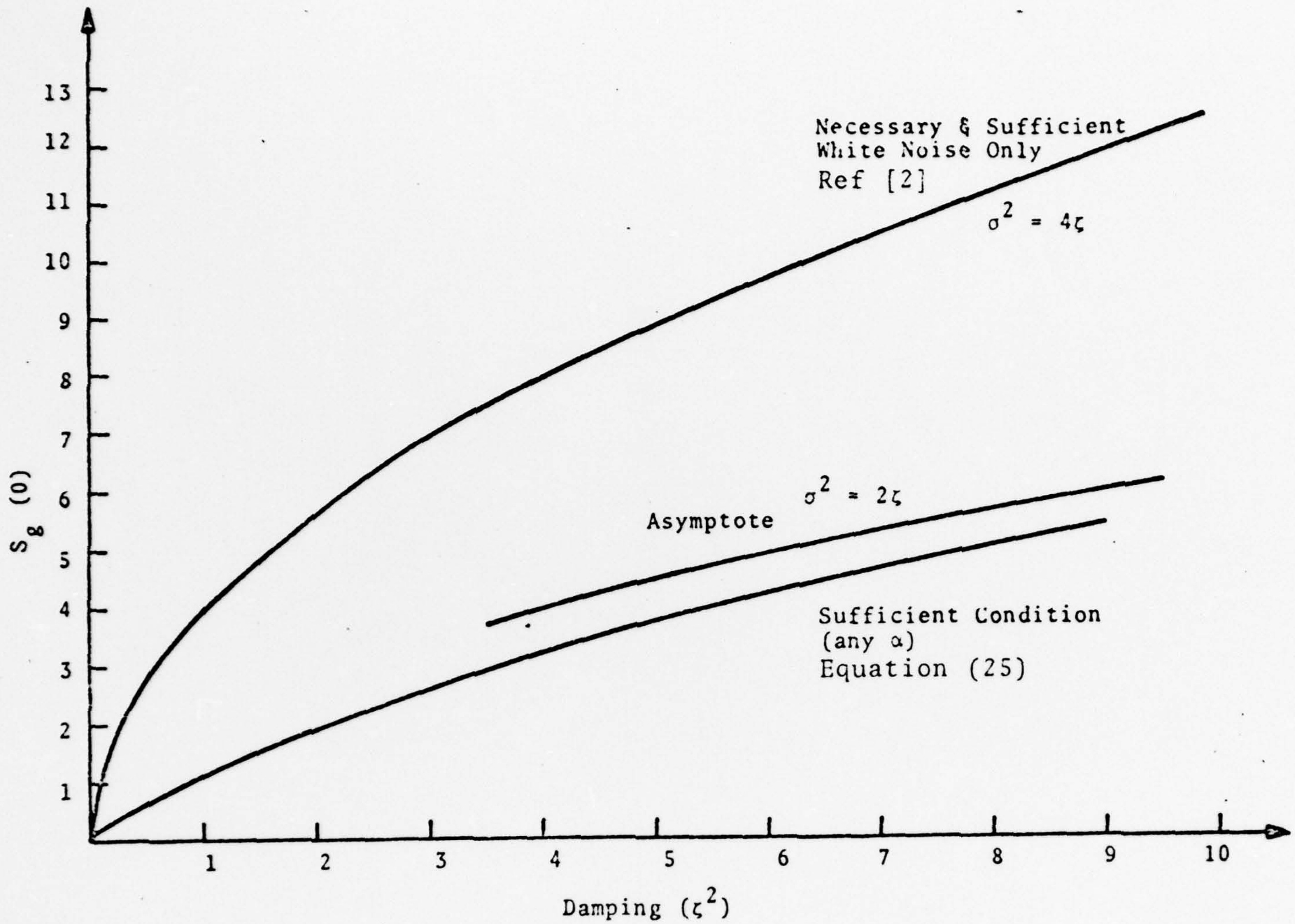

FIGURE 4

DATA FROM FIGURE 3 REPLOTTED FOR COMPARISON TO WHITE NOISE CASE quite conservative. We remark in passing that the result of Infante can not be superimposed onto Figure (4) because his result would appear as a straight line whose slope is proportional to α^{-1}. So as $\alpha \to \infty$, Infante's boundary would approach the horizontal axis.

Clearly, the idea of using the results of Section III for bounding the mean square response of higher order systems applies to more than just second order systems. It works for any single-input, single-output system whose impulse response can be bounded by a decaying exponential, and whose noisy parameter can be treated as a feedback gain.

V. CONCLUSION:

We have demonstrated that the bandwidth of the colored noise process is an important consideration in determining the stability properties of System (15), and provided a boundary for the known region of stability that explicitly involves the bandwidth of the colored noise. For large (but finite) bandwidths and for small damping, the results enlarged the previously known region of stability for the system considered. Also, the new bound was derived in such a way that it remains valid in the limit as the colored noise parameter becomes white. Any bound that depends only on the mean square value of the noise process will necessarily fail as the bandwidth becomes arbitrarily large.

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Uncertain elements must be considered in the mathematical model of many dynamical systems. The thoery associated with additive noise models is quite advanced, but many control problems are more realistically modeled as containing uncertain and time varying parameters or gains. For multiplicative gains of the white noise type, necessary and sufficient conditions for the stability of many of these systems have been derived previously. In this article, we develop conditions for the stability of some continuous-time systems containing multiplicative colored noise.

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